MATHEMATICS

1. Suppose a, b, $c \in R$ and $b \neq c$. If α, β are roots of $x^2 + ax + b = 0$ and γ, δ are roots of $x^2 + ax + c = 0$, then the equation whose roots are $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$ and 2 is

(a) $x^2 + x + 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 - 3x + 2 = 0$ (d) $x^2 - 3x - 2 = 0$

2. The number of ways in which we can arrange the digits 1, 2, 3,....., 9 such that the product of five digits at any of the five consecutive position is divisible by 7 is

(a) 7! (b) ${}^9P_7$
(c) 8! (d) 5(7!)

3. Let a_n be the n^{th} term of an A.P. with common difference d. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then $(\alpha - \beta) - 100d =$

(a) 0 (b) 50
(c) 100 (d) None of these

4. If $f(x) = x^{2/3}$, then

(a) (0, 0) is a point of maximum
(b) (0, 0) is not a point of minimum
(c) (0, 0) is a critical point
(d) There is no critical point

5. The solution of $\frac{dy}{dx} = \frac{1}{2x - y^2}$ is given by

(a) $y = Ce^{-2x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$

(b) $x = Ce^{-y} + \frac{1}{4}y^2 + \frac{1}{4}y + \frac{1}{2}$

(c) $x = Ce^{y} + \frac{1}{4}y^2 + y + \frac{1}{2}$

(d) $x = Ce^{2y} + \frac{1}{2}y^2 + \frac{1}{2}y + \frac{1}{4}$

6. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola, then eccentricity 'e' of the hyperbola satisfied

(a) $e = \frac{2}{\sqrt{3}}$ (b) $e = \frac{\sqrt{3}}{2}$

(c) $e > \frac{2}{\sqrt{3}}$ (d) $1 < e < \frac{2}{\sqrt{3}}$

7. The value of k for which the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$ is

(a) 7 (b) 6
(c) no real value (d) – 7

8. Each of two persons A and B toss three fair coins. The probability that both get the same number of heads is

(a) $\frac{3}{8}$ (b) $\frac{1}{9}$

(c) $\frac{5}{16}$ (d) None of these

9. A tower is standing at the centre of an elliptic field. If Aditya observes that the angle of elevation on the top of the tower at an extremity of the major axis of the field is α, at it focus is β and and an extremity of the minor axis is γ, then

(a) $\cot^2\alpha = \cot^2\beta - \cot^2\gamma$
(b) $\cot^2\beta = \cot^2\gamma - \cot^2\alpha$
(c) $\cot^2\gamma = \cot^2\alpha - \cot^2\beta$
(d) None of these

10. Let $f(x) \geq 0$ $\forall$ $x \geq 0$ be a non-negative continuous function. If $f'(x)\cos x \leq f(x)\sin x$ $\forall$ $x \geq 0$, then the value of $f\left(\frac{5\pi}{3}\right)$ is

(a) $e^{\frac{-1}{\sqrt{3}}}$ (b) $\frac{\sqrt{3}-1}{2}$

(c) $\frac{\sqrt{3}+1}{2}$ (d) 0

11. If $(f(x))^2 f\left(\frac{1-x}{1+x}\right) = x^3, x \neq -1, 1$ and $f(x) \neq 0$, then $f(x)$ is

(a) $x^2\left(\frac{1-x}{1+x}\right)$ (b) $x^2\left(\frac{1+x}{1-x}\right)$

(c) $x^2\left(\frac{1-2x}{1+x}\right)$ (d) None of these

12. Let $\Delta(x)=\begin{vmatrix}\sin x & \cos x & \sin 2x+\cos 2x\\ 0 & 1 & 1\\ 1 & 0 & -1\end{vmatrix}$ then

$\Delta'(x)$ vanishes at least once in

(a) $(0, \pi/2)$ (b) $(\pi/2, \pi)$
(c) $(0, \pi/4)$ (d) $(-\pi/2, 0)$

13. Coefficient of x^{17} in the polynomial

$P(x)=\prod_{r=0}^{17}(x+ {}^{35}C_r)$ is

(a) 2^{34} (b) ${}^{36}C_{17}$
(c) $2^{35}-{}^{36}C_{17}$ (d) 0

14. If $\lim_{x\to 0}\frac{(\cos x)^{1/2}-(\cos x)^{1/3}}{\sin^2 x}=a$ then the value of $12a+2$ is

(a) 1 (b) – 1
(c) 2 (d) None of these

15. The difference between the greatest and least values of the function

$f(x)=\cos x+\left(\frac{1}{2}\right)\cos 2x-\left(\frac{1}{3}\right)\cos 3x$ is

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$
(c) $\frac{8}{7}$ (d) $\frac{9}{4}$

16. The incentre of triangle with vertices $\left(-\sqrt{3},-1\right)$ (0, 0) and (0, – 2) is

(a) $\left(-\sqrt{3},-1\right)$ (b) $\left(-\frac{1}{\sqrt{3}},-1\right)$
(c) $\left(-\frac{2}{\sqrt{3}},-1\right)$ (d) None of these

17. The slope of the line touching both the parabolas $y^2=4x$ and $x^2=-32y$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{1}{8}$ (d) $\frac{2}{3}$

18. If a variable take the value of 0, 1, 2,......,n with frequencies proportional to the binomial coefficients ${}^nC_0$, ${}^nC_1$,...... ${}^nC_n$ then the mean of the distribution is

(a) $\frac{n(n+1)}{4}$ (b) $\frac{n}{2}$
(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

19. If $\cos A=\frac{3}{4}$, then the value of

$16\cos^2\left(\frac{A}{2}\right)-32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)$ is

(a) – 4 (b) – 3
(c) 3 (d) 4

20. If $n\in N$, then the remainder when $37^{n+2}+16^{n+1}+30^n$ is divided by 7 is

(a) 0 (b) 1
(c) 2 (d) 5